Plane_Edge_SLAM

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OUTLINE

1. Plane Parameter Estimation
   - Least Squares
   - weighted LS
2. RGB-D Camera Pose Estimation
   - Constraint Analysis

OUTLINE

- 平面参数估计
  - 最小二乘方法
  - 考虑测量误差的最小二乘方法
  - 考虑测量误差以及入射角的最小二乘方法
  - 平面参数误差的估计
- RGB-D传感器位姿估计
  - 匹配的平面对位姿估计的约束分析
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  - 对应关系的建立
- 实验结果

平面参数估计

最小二乘方法

已知构成平面的点集$\{\mathbf{p}_{\pi j}\}_{j=1,\cdots,N_{p\pi}}$(由平面提取方法得到)，求解平面参数$\mathbf{n},d$ 使得点到平面的距离平方和最小。

$$F(\mathbf{n},d)=\sum_{j=1}^{N_{p\pi}} D^2(\mathbf{p}_{\pi j},\mathbf{n},d)=\sum_{j=1}^{N_{p\pi}}\left(\mathbf{n}^T\mathbf{p}_{\pi j}+d\right)^2 \tag{1}$$

$$(\mathbf{n}^*,d^*)=\arg\min_{(\mathbf{n},d)} F(\mathbf{n},d) \tag{2}$$

求F对d的偏导数并令其为0可得

$$d^*=-\mathbf{n}^T\mathbf{p}_G \tag{3}$$

其中$\mathbf{p}_G$是点的重心位置。

$$\mathbf{p}_G=\frac{1}{N_{p\pi}}\sum_{j=1}^{N_{p\pi}}\mathbf{p}_{\pi j} \tag{4}$$

平面参数估计

最小二乘方法

将式(3)代入式(1)可得

$$\begin{aligned} F(\mathbf{n}) &= \sum_{j=1}^{N_{p\pi}} \left(\mathbf{n}^T (\mathbf{p}_{\pi j} - \mathbf{p}_G)\right)^2 \\ &= \mathbf{n}^T \sum_{j=1}^{N_{p\pi}} (\mathbf{p}_{\pi j} - \mathbf{p}_G)(\mathbf{p}_{\pi j} - \mathbf{p}_G)^T \mathbf{n} \end{aligned} \tag{5}$$

令

$$\mathbf{S} = \sum_{j=1}^{N_{p\pi}} (\mathbf{p}_{\pi j} - \mathbf{p}_G)(\mathbf{p}_{\pi j} - \mathbf{p}_G)^T \tag{6}$$

则有

$$\mathbf{n}^* = \arg\min_{\mathbf{n}} \mathbf{n}^T \mathbf{S} \mathbf{n} \tag{7}$$

式(7)的解$\mathbf{n}^*$即为矩阵$\mathbf{S}$的最小特征值对应的特征向量。

平面参数估计

考虑测量误差的最小二乘方法

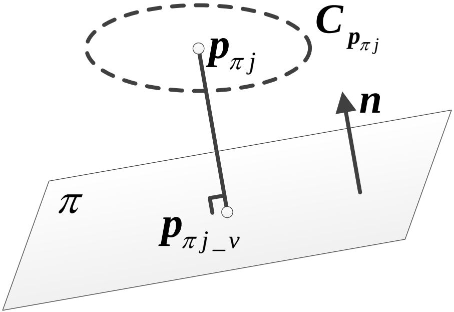


Figure: 考虑误差时点到平面的距离。

平面参数估计

考虑测量误差的最小二乘方法

假设点$\mathbf{p}_{\pi j}$的测量误差可以用协方差矩阵$\mathbf{C}_{\mathbf{p}_{\pi j}}$来表示[1]。点$\mathbf{p}_{\pi j}$在平面$\pi$上的垂足为$\mathbf{p}_{\pi j_v}$。

$$\mathbf{p}_{\pi j\_v} = \mathbf{p}_{\pi j} - \left(\mathbf{n}^T \mathbf{p}_{\pi j} + d\right) \mathbf{n} \tag{8}$$

计算点$\mathbf{p}_{\pi j}$到$\mathbf{p}_{\pi j_v}$的马氏距离作为点到平面的距离$D\left(\mathbf{p}_{\pi j}, \mathbf{n}, d\right)$，如图1所示。

$$\begin{aligned} D^2\left(\mathbf{p}_{\pi j}, \mathbf{n}, d\right) &= \left(\mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_v}\right)^T \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \left(\mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_v}\right) \\ &= \left(\mathbf{n}^T \mathbf{p}_{\pi j} + d\right)^2 \mathbf{n}^T \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \mathbf{n} \end{aligned} \tag{9}$$

令

$$c_j(\mathbf{n}) = \mathbf{n}^T \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \mathbf{n} \tag{10}$$

[1] Fast visual odometry and mapping from RGB-D data, ICRA, 2013

平面参数估计

考虑测量误差的最小二乘方法

令

$$\mathbf{v}_{\mathbf{p}\pi j} = \mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_v} \tag{11}$$

则

$$\mathbf{C}_{v\mathbf{p}\pi j} = \left(\frac{\partial v_{\mathbf{p}\pi j}}{\partial \mathbf{p}_{\pi j}}\right) \mathbf{C}_{\mathbf{p}_{\pi j}} \left(\frac{\partial v_{\mathbf{p}\pi j}}{\partial \mathbf{p}_{\pi j}}\right)^T \tag{12}$$

$$\frac{\partial v_{\mathbf{p}\pi j}}{\partial \mathbf{p}_{\pi j}} = \mathbf{I}_{3\times 3} - \mathbf{n}\mathbf{n}^T \tag{13}$$

$$\begin{aligned} D^2(\mathbf{p}_{\pi j}, \mathbf{n}, d) &= (\mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_v})^T \mathbf{C}_{v\mathbf{p}\pi j}^{-1} (\mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_v}) \\ &= (\mathbf{n}^T \mathbf{p}_{\pi j} + d)^2 \mathbf{n}^T \mathbf{C}_{v\mathbf{p}\pi j}^{-1} \mathbf{n} \end{aligned} \tag{14}$$

平面参数估计

考虑测量误差的最小二乘方法

$$F\left(\mathbf{n},d\right)=\sum_{j=1}^{N_{p\pi}}D^2\left(\mathbf{p}_{\pi j},\mathbf{n},d\right)=\sum_{j=1}^{N_{p\pi}}c_j(\mathbf{n})\left(\mathbf{n}^T\mathbf{p}_{\pi j}+d\right)^2 \tag{15}$$

$$c_j(\mathbf{n})=\mathbf{n}^T\mathbf{C}_{\mathbf{p}_{\pi j}}^{-1}\mathbf{n} \tag{16}$$

$$\left(\mathbf{n}^*,d^*\right)=\arg\min_{(\mathbf{n},d)}F\left(\mathbf{n},d\right) \tag{17}$$

这里作一个近似，令$c_j(\mathbf{n})=c_j(\hat{\mathbf{n}})$，$\hat{\mathbf{n}}$表示式(7)的解，即不考虑测量误差时的法向量估计结果。求F对d的偏导数并令其为0可得

$$d^*=-\mathbf{n}^T\mathbf{p}_G \tag{18}$$

其中

$$\mathbf{p}_G=\frac{\sum_{j=1}^{N_{p\pi}}c_j(\hat{\mathbf{n}})\mathbf{p}_{\pi j}}{\sum_{j=1}^{N_{p\pi}}c_j(\hat{\mathbf{n}})} \tag{19}$$

平面参数估计

考虑测量误差的最小二乘方法

将式(18)代入(15)可得

$$\begin{aligned} F(\mathbf{n}) &= \mathbf{n}^T \sum_{j=1}^{N_{p\pi}} c_j(\hat{\mathbf{n}})(\mathbf{p}_{\pi j} - \mathbf{p}_G)(\mathbf{p}_{\pi j} - \mathbf{p}_G)^T \mathbf{n} \\ &= \mathbf{n}^T \mathbf{S} \mathbf{n} \end{aligned} \tag{20}$$

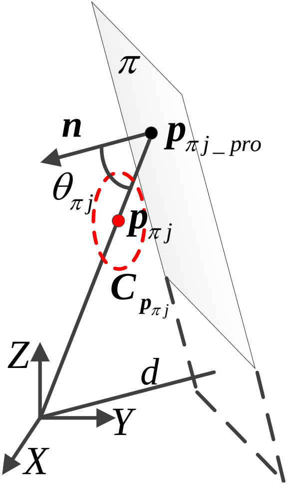
则

$$\mathbf{n}^* = \arg\min_{\mathbf{n}} F(\mathbf{n}) = \arg\min_{\mathbf{n}} \mathbf{n}^T \mathbf{S} \mathbf{n} \tag{21}$$

式(21)的解$\mathbf{n}^*$即为矩阵$\mathbf{S}$的最小特征值对应的特征向量。

平面参数估计

考虑测量误差以及入射角的最小二乘方法



平面参数估计

考虑测量误差以及入射角的最小二乘方法

由坐标系原点到$\mathbf{p}_{\pi j}$的射线与平面π的交点为$\mathbf{p}_{\pi j_pro}$，如图3所示。

$$\mathbf{p}_{\pi j\_pro} = \frac{-d}{\mathbf{n}^T \mathbf{p}_{\pi j}} \mathbf{p}_{\pi j} \tag{22}$$

计算点$\mathbf{p}_{\pi j}$到$\mathbf{p}_{\pi j_pro}$的马氏距离作为点到平面的距离$D\left(\mathbf{p}_{\pi j}, \mathbf{n}, d\right)$。

$$\begin{aligned} D^2\left(\mathbf{p}_{\pi j}, \mathbf{n}, d\right) &= \left(\mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_pro}\right)^T \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \left(\mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_pro}\right) \\ &= \left(\mathbf{n}^T \mathbf{p}_{\pi j} + d\right)^2 \frac{\mathbf{p}_{\pi j}^T}{\mathbf{n}^T \mathbf{p}_{\pi j}} \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \frac{\mathbf{p}_{\pi j}}{\mathbf{n}^T \mathbf{p}_{\pi j}} \end{aligned} \tag{23}$$

平面参数估计

考虑测量误差以及入射角的最小二乘方法

令

$$v_{\mathbf{p}\pi j} = \mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_pro} \tag{24}$$

则

$$\mathbf{C}_{v\mathbf{p}\pi j} = \left(\frac{\partial v_{\mathbf{p}\pi j}}{\partial \mathbf{p}_{\pi j}}\right) \mathbf{C}_{\mathbf{p}_{\pi j}} \left(\frac{\partial v_{\mathbf{p}\pi j}}{\partial \mathbf{p}_{\pi j}}\right)^T \tag{25}$$

$$\frac{\partial v_{\mathbf{p}\pi j}}{\partial \mathbf{p}_{\pi j}} = \left(1 + \frac{d}{\mathbf{n}^T \mathbf{p}_{\pi j}}\right) \mathbf{I}_{3\times 3} - \frac{d}{\left(\mathbf{n}^T \mathbf{p}_{\pi j}\right)^2} \mathbf{p}_{\pi j} \mathbf{n}^T \tag{26}$$

$$\begin{aligned} D^2\left(\mathbf{p}_{\pi j}, \mathbf{n}, d\right) &= \left(\mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_pro}\right)^T \mathbf{C}_{v\mathbf{p}\pi j}^{-1} \left(\mathbf{p}_{\pi j} - \mathbf{p}_{\pi j\_pro}\right) \\ &= \left(\mathbf{n}^T \mathbf{p}_{\pi j} + d\right)^2 \frac{\mathbf{p}_{\pi j}^T}{\mathbf{n}^T \mathbf{p}_{\pi j}} \mathbf{C}_{v\mathbf{p}\pi j}^{-1} \frac{\mathbf{p}_{\pi j}}{\mathbf{n}^T \mathbf{p}_{\pi j}} \end{aligned} \tag{27}$$

平面参数估计

考虑测量误差以及入射角的最小二乘方法

令

$$c_j(\mathbf{n}) = \frac{\mathbf{p}_{\pi j}^T}{\mathbf{n}^T \mathbf{p}_{\pi j}} \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \frac{\mathbf{p}_{\pi j}}{\mathbf{n}^T \mathbf{p}_{\pi j}} \tag{28}$$

考虑$c_j(\mathbf{n})$中各项，向量$\frac{\mathbf{p}_{\pi j}^T}{\mathbf{n}^T \mathbf{p}_{\pi j}}$的方向即为向量$\mathbf{p}_{\pi j}$的方向，设其方向的单位向量为$\mathbf{v}_{\mathbf{p}_{\pi j}}$，其幅值为

$$\left| \frac{\mathbf{p}_{\pi j}^T}{\mathbf{n}^T \mathbf{p}_{\pi j}} \right| = \frac{1}{\cos \theta_{\pi j}} \tag{29}$$

其中$\theta_{\pi j} \in [0, \frac{\pi}{2})$为向量$\mathbf{p}_{\pi j}$向平面$\pi$的入射角，如图3 中所示。则有

$$c_j(\mathbf{n}) = \frac{1}{\cos^2 \theta_{\pi j}} \mathbf{v}_{\mathbf{p}_{\pi j}}^T \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \mathbf{v}_{\mathbf{p}_{\pi j}} \tag{30}$$

平面参数估计

考虑测量误差以及入射角的最小二乘方法

$$c_j(\mathbf{n}) = \frac{1}{\cos^2\theta_{\pi j}} \mathbf{v}_{\mathbf{p}_{\pi j}}^T \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \mathbf{v}_{\mathbf{p}_{\pi j}} \quad (31)$$

其中$\mathbf{v}_{\mathbf{p}_{\pi j}}^T \mathbf{C}_{\mathbf{p}_{\pi j}}^{-1} \mathbf{v}_{\mathbf{p}_{\pi j}}$即为协方差矩阵在$\mathbf{v}_{\mathbf{p}_{\pi j}}$方向的分量，而$\frac{1}{\cos^2\theta_{\pi j}}$这一项与入射角$\theta_{\pi j}$相关，$\theta_{\pi j}$越大，$c_j(\mathbf{n})$越大，则距离$D_{w2}(\mathbf{p}_{\pi j}, \mathbf{n}, d)$也越大。同样的，这里做一个近似，令$\mathbf{n} = \hat{\mathbf{n}}$，$\hat{\mathbf{n}}$表示式(7)的解，这时有$c_j(\mathbf{n}) = c_j(\hat{\mathbf{n}})$。

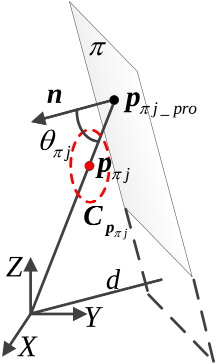


Figure: 考虑误差以及入射角时点到平

平面参数估计

考虑测量误差以及入射角的最小二乘方法

则

$$F(\mathbf{n}, d) = \sum_{j=1}^{N_{p\pi}} D^2(\mathbf{p}_{\pi j}, \mathbf{n}, d) = \sum_{j=1}^{N_{p\pi}} c_j(\hat{\mathbf{n}}) \left(\mathbf{n}^T \mathbf{p}_{\pi j} + d\right)^2 \tag{32}$$

$$(\mathbf{n}^*, d^*) = \arg\min_{(\mathbf{n}, d)} F(\mathbf{n}, d) \tag{33}$$

求F对d的偏导数并令其为0可得

$$d^* = -\mathbf{n}^T \mathbf{p}_G \tag{34}$$

其中

$$\mathbf{p}_G = \frac{\sum_{j=1}^{N_{p\pi}} c_j(\hat{\mathbf{n}}) \mathbf{p}_{\pi j}}{\sum_{j=1}^{N_{p\pi}} c_j(\hat{\mathbf{n}})} \tag{35}$$

平面参数估计

考虑测量误差以及入射角的最小二乘方法

将式(34)代入(32)可得

$$\begin{aligned} F(\mathbf{n}) &= \mathbf{n}^T \sum_{j=1}^{N_{p\pi}} c_j(\hat{\mathbf{n}}) (\mathbf{p}_{\pi j} - \mathbf{p}_G)(\mathbf{p}_{\pi j} - \mathbf{p}_G)^T \mathbf{n} \\ &= \mathbf{n}^T \mathbf{S} \mathbf{n} \end{aligned} \quad (36)$$

则有

$$\mathbf{n}^* = \arg\min_{\mathbf{n}} \mathbf{n}^T \mathbf{S} \mathbf{n} \quad (37)$$

式(37)的解$\mathbf{n}^*$即为矩阵$\mathbf{S}$的最小特征值对应的特征向量。

平面参数估计

平面参数误差的估计

$$F(\mathbf{n},d)=\sum_{j=1}^{N_{p\pi}} D^2(\mathbf{p}_{\pi j},\mathbf{n},d)=\sum_{j=1}^{N_{p\pi}} c_j(\hat{\mathbf{n}})\left(\mathbf{n}^T\mathbf{p}_{\pi j}+d\right)^2 \tag{38}$$

$$(\mathbf{n}^*,d^*)=\arg\min_{(\mathbf{n},d)} F(\mathbf{n},d) \tag{39}$$

计算(38)在$\mathbf{n}^*,d^*$处的Hessian矩阵，并取其逆作为平面参数协方差的估计[2]。

$$\begin{aligned}\mathbf{C}_\pi^{-1}=\mathbf{H}_\pi&=\begin{bmatrix}\mathbf{H}_{\mathbf{nn}} & \mathbf{H}_{\mathbf{n}d}\\ \mathbf{H}_{\mathbf{n}d}^T & \mathbf{H}_{dd}\end{bmatrix}\\ &=\begin{bmatrix}\frac{\partial^2F}{\partial\mathbf{n}^2} & \left(\frac{\partial^2F}{\partial d\partial\mathbf{n}}\right)\\ \left(\frac{\partial^2F}{\partial d\partial\mathbf{n}}\right)^T & \frac{\partial^2F}{\partial d^2}\end{bmatrix}=\sum_{j=1}^{N_{p\pi}} c_j\begin{bmatrix}\mathbf{p}_{\pi j}\mathbf{p}_{\pi j}^T & \mathbf{p}_{\pi j}\\ \mathbf{p}_{\pi j}^T & 1\end{bmatrix}\end{aligned} \tag{40}$$

[2] K. Pathak, N. Vaskevicius and A. Birk, Uncertainty analysis for optimum plane extraction from noisy 3D range-sensor point-clouds. Intelligent Service

RGB-D传感器位姿估计

i subscript	平面下标;
k subscript	边缘点下标;
c presuperscript	当前帧;
r presuperscript	参考帧;
$\pi = [\mathbf{n}^T, d]^T$	平面参数;
$\{{}^c\pi_i, {}^r\pi_i\}_{i=1,\cdots,N_\pi}$	两帧之间的对应平面;
$\{{}^c\mathbf{p}_k, {}^r\mathbf{p}_k\}_{k=1,\cdots,N_p}$	两帧之间的对应边缘点;
$\begin{bmatrix} \mathbf{R}_{cr} & \mathbf{t}_{cr} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix} \in \mathbb{SE}(3)$	参考帧到当前帧的变换矩阵;
$\xi = [\mathbf{t}^T, \omega^T]^T \in \mathfrak{se}(3)$	6-DoF位姿变换;

RGB-D传感器位姿估计

匹配的平面对位姿估计的约束分析

假设连续两帧(当前帧和参考帧)之间有N_π对匹配平面$\{{}^c\pi_i, {}^r\pi_i\}_{i=1,\cdots,N_\pi}$ (平面间对应关系已建立，详见33)。定义目标函数

$$J_\pi(\xi) = \sum_{i=1}^{N_\pi} J_{\pi i}(\xi) \tag{41}$$

$$J_{\pi i}(\xi) = \frac{1}{2}\left({}^c\pi_i - T_{cr}\left({}^r\pi_i, \xi\right)\right)^T {}^c\mathbf{C}_{\pi i}^{-1}\left({}^c\pi_i - T_{cr}\left({}^r\pi_i, \xi\right)\right) \tag{42}$$

$$T_{cr}\left({}^r\pi_i, \xi\right) = \begin{bmatrix} \mathbf{R}_{cr}{}^r\mathbf{n}_i \\ {}^rd_i - \left(\mathbf{R}_{cr}{}^r\mathbf{n}_i\right)^T \mathbf{t}_{cr} \end{bmatrix} \tag{43}$$

RGB-D传感器位姿估计

匹配的平面对位姿估计的约束分析

$$v_{\pi i} = {}^c\pi_i - T_{cr}({}^r\pi_i, \xi) \tag{44}$$

$$\mathbf{C}_{v\pi i} = {}^c\mathbf{C}_{\pi i} + \left(\frac{\partial v_{\pi i}}{\partial {}^r\pi_i}\right) {}^r\mathbf{C}_{\pi i} \left(\frac{\partial v_{\pi i}}{\partial {}^r\pi_i}\right)^T \tag{45}$$

$$\frac{\partial v_{\pi i}}{\partial {}^r\pi_i} = \begin{bmatrix} -\mathbf{R}_{cr} & \mathbf{0}_{3\times 1} \\ \mathbf{t}_{cr}^T \mathbf{R}_{cr} & -1 \end{bmatrix} \tag{46}$$

$$J_{\pi i}(\xi) = \frac{1}{2}({}^c\pi_i - T_{cr}({}^r\pi_i, \xi))^T \, {}^c\mathbf{C}_{v\pi i}^{-1}({}^c\pi_i - T_{cr}({}^r\pi_i, \xi)) \tag{47}$$

RGB-D传感器位姿估计

匹配的平面对位姿估计的约束分析

求解目标函数中一项$J_{\pi i}$在ξ处的梯度可得

$$\frac{\partial J_{\pi i}}{\partial \xi} = \begin{bmatrix} h_{di} \cdot \mathbf{R}_{cr}{}^{r}\mathbf{n}_i \\ \left(\mathbf{R}_{cr}{}^{r}\mathbf{n}_i\right)_{\times} \left(h_{di} \cdot \mathbf{t}_{cr} - \mathbf{h}_{\mathbf{n}i}\right) \end{bmatrix} \tag{48}$$

其中$\left(\mathbf{R}_{cr}{}^{r}\mathbf{n}_i\right)_{\times}$是$\mathbf{R}_{cr}{}^{r}\mathbf{n}_i$对应的反对称矩阵。

$$\begin{aligned} \mathbf{h}_{\mathbf{n}i} &= \mathbf{H}_{\mathbf{nn}i} \cdot \triangle \mathbf{n}_i + \mathbf{H}_{\mathbf{n}di} \cdot \triangle d_i \\ h_{di} &= \mathbf{H}_{\mathbf{n}di}^{T} \cdot \triangle \mathbf{n}_i + \mathbf{H}_{ddi} \cdot \triangle d_i \\ \triangle \mathbf{n}_i &= {}^{c}\mathbf{n}_i - \mathbf{R}_{cr}{}^{r}\mathbf{n}_i \\ \triangle d_i &= {}^{r}d_i - \left({}^{r}d_i - \left(\mathbf{R}_{cr}{}^{r}\mathbf{n}_i\right)^{T} \mathbf{t}_{cr}\right) \end{aligned} \tag{49}$$

RGB-D传感器位姿估计

匹配的平面对位姿估计的约束分析

令Ψ_π为$\{\frac{\partial J_{\pi i}}{\partial \xi}\}_{i=1,\cdots,N_\pi}$的散列矩阵。

$$\Psi_\pi = \sum_{i=1}^{N_\pi} \left(\frac{\partial J_{\pi i}}{\partial \xi}\right)\left(\frac{\partial J_{\pi i}}{\partial \xi}\right)^T = \sum_{i=1}^{N_\pi} \begin{bmatrix} \Psi_{\pi 11} & \Psi_{\pi 12} \\ \Psi_{\pi 21} & \Psi_{\pi 22} \end{bmatrix} \tag{50}$$

其中

$$\begin{aligned} \Psi_{\pi 11} &= h_{di}^2 \cdot (\mathbf{R}_{cr}{}^r\mathbf{n}_i)(\mathbf{R}_{cr}{}^r\mathbf{n}_i)^T \\ \Psi_{\pi 12} &= h_{di} \cdot (\mathbf{R}_{cr}{}^r\mathbf{n}_i)(h_{di} \cdot \mathbf{t}_{cr} - \mathbf{h}_{\mathbf{n}i})^T (\mathbf{R}_{cr}{}^r\mathbf{n}_i)_\times^T \\ \Psi_{\pi 21} &= h_{di} \cdot (\mathbf{R}_{cr}{}^r\mathbf{n}_i)_\times (h_{di} \cdot \mathbf{t}_{cr} - \mathbf{h}_{\mathbf{n}i})(\mathbf{R}_{cr}{}^r\mathbf{n}_i)^T \\ \Psi_{\pi 22} &= (\mathbf{R}_{cr}{}^r\mathbf{n}_i)_\times (h_{di} \cdot \mathbf{t}_{cr} - \mathbf{h}_{\mathbf{n}i})(h_{di} \cdot \mathbf{t}_{cr} - \mathbf{h}_{\mathbf{n}i})^T (\mathbf{R}_{cr}{}^r\mathbf{n}_i)_\times^T \end{aligned} \tag{51}$$

RGB-D传感器位姿估计

匹配的平面对位姿估计的约束分析

The matrix Ψ_π is actually a scatter matrix which contains information about the distribution of the gradient of $J_{\pi,i}$ w.r.t. $\mathbf{w}$ over all planes in the matched plane set. Performing principal component analysis upon Ψ_π results in

$$\Psi_\pi = Q_\pi \Lambda_\pi Q_\pi^T = \sum_{l=1}^{6} \lambda_{\pi l} \mathbf{q}_{\pi l} \mathbf{q}_{\pi l} \tag{52}$$

where $\lambda_{\pi 1} \geq \lambda_{\pi 2} \geq \cdots \geq \lambda_{\pi 6}$ are the eigenvalues of Ψ_π, and $\mathbf{q}_{\pi l}$ are the corresponding eigenvectors, of which the first three elements are the translation components, and the last three elements are the rotation components. The eigenvector $\mathbf{q}_{\pi l}$ corresponding to the largest eigenvalue represents the transformation of maximum constraint. Perturbing the plane parameters by the transformation of the direction $\mathbf{q}_{\pi l}$ will result in the largest possible change in from among all possible transformation perturbations.

RGB-D传感器位姿估计

基于平面的位姿估计的两种退化情况

$$\Psi_\pi |_{\xi=0} = \sum_{i=1}^{N_\pi} \begin{bmatrix} h_{di}^2 \cdot {}^r\mathbf{n}_i {}^r\mathbf{n}_i^T & -h_{di} \cdot {}^r\mathbf{n}_i \mathbf{h}_{\mathbf{n}i}^T {}^r\mathbf{n}_{i\times}^T \\ -h_{di} \cdot {}^r\mathbf{n}_{i\times} \mathbf{h}_{\mathbf{n}i} {}^r\mathbf{n}_i^T & {}^r\mathbf{n}_{i\times} \mathbf{h}_{\mathbf{n}i} \mathbf{h}_{\mathbf{n}i}^T {}^r\mathbf{n}_{i\times}^T \end{bmatrix} \tag{53}$$

Define the matrix $\mathbf{M}$ and compute its SVD decomposition as

$$\mathbf{M} = \sum_{i=1}^{N_\pi} {}^r\mathbf{n}_i {}^c\mathbf{n}_i^T = \mathbf{U}\Lambda\mathbf{V}^T = \lambda_1 \mathbf{u}_1 \mathbf{v}_1^T + \lambda_2 \mathbf{u}_2 \mathbf{v}_2^T + \lambda_3 \mathbf{u}_3 \mathbf{v}_3^T \tag{54}$$

where the singular values $\lambda_1, \lambda_2, \lambda_3$ satisfy $\lambda_1 \geq \lambda_2 \geq \lambda_3$.

RGB-D传感器位姿估计

基于平面的位姿估计的两种退化情况

(1) Assuming that $\lambda_3 = 0$, ${}^r\mathbf{n}_i^T\mathbf{u}_3 = 0$ holds true for all $i = 1, \cdots, N_\pi$. For a small camera motion $\triangle\xi = [\mu\mathbf{u}_3^T, \mathbf{0}^T]^T$ in the direction of $\mathbf{u}_3$, the variation of the cost function $\triangle J_\pi^2(\triangle\xi) = \triangle\xi^T\Psi_\pi\triangle\xi$ caused by $\triangle\xi$ is always zero. That is to say, the perturbation in the direction of $\mathbf{u}_3$ will cause no change of the cost function.

(2) Similarly, when $\lambda_2 = \lambda_3 = 0$, for all $i = 1, \cdots, N_\pi$, ${}^r\mathbf{n}_i$ satisfies ${}^r\mathbf{n}_i^T\mathbf{u}_2 = 0$, ${}^r\mathbf{n}_i^T\mathbf{u}_3 = 0$ and ${}^r\mathbf{n}_i \times \mathbf{u}_1 = 0$. In this case, for a small camera motion $\triangle\xi = [\mu_2\mathbf{u}_2^T + \mu_3\mathbf{u}_3^T, \mu_1\mathbf{u}_1^T]^T$, $\triangle J_\pi^2(\triangle\xi) = 0$.

RGB-D传感器位姿估计

基于平面和边缘点的位姿估计

假设连续两帧(当前帧和参考帧)之间有N_p对匹配的边缘点$\{{}^c\mathbf{p}_k, {}^r\mathbf{p}_k\}_{k=1,\cdots,N_p}$ (假设对应关系已知，详见33)。
定义基于平面和边缘点的目标函数为

$$J(\xi) = J_\pi(\xi) + W_p \sum_{k=1}^{N_p} w_{pk} J_{pk}(\xi) \quad (55)$$

其中$J_\pi(\xi)$定义如式(41)，

$$J_{pk}(\xi) = \frac{1}{2}\left({}^c\mathbf{p}_k - T_{cr}({}^r\mathbf{p}_k, \xi)\right)^T {}^c\mathbf{C}_{pk}^{-1}\left({}^c\mathbf{p}_k - T_{cr}({}^r\mathbf{p}_k, \xi)\right) \quad (56)$$

$$T_{cr}({}^r\mathbf{p}_k, \xi) = \mathbf{R}_{cr}{}^r\mathbf{p}_k + \mathbf{t}_{cr} \quad (57)$$

${}^c\mathbf{C}_{pk}$为边缘点${}^c\mathbf{p}_k$的协方差，其估计方法下文详述。
参数W_p与w_{pk}的计算方法下文详述。

RGB-D传感器位姿估计

基于平面和边缘点的位姿估计

$$v_{pk} = {}^{c}\mathbf{p}_k - T_{cr}({}^{r}\mathbf{p}_k, \xi) \tag{58}$$

$$\mathbf{C}_{vpk} = {}^{c}\mathbf{C}_{pk} + \mathbf{R}_{cr}{}^{r}\mathbf{C}_{pk}\mathbf{R}_{cr}^{T} \tag{59}$$

$$J_{pk}(\xi) = \frac{1}{2}({}^{c}\mathbf{p}_k - T_{cr}({}^{r}\mathbf{p}_k, \xi))^{T}\, {}^{c}\mathbf{C}_{vpk}^{-1}({}^{c}\mathbf{p}_k - T_{cr}({}^{r}\mathbf{p}_k, \xi)) \tag{60}$$

RGB-D传感器位姿估计

基于平面和边缘点的位姿估计

${}^c\mathbf{C}_{pk}$的估计

取${}^c\mathbf{p}_k$邻域内的边缘点，拟合协方差${}^c\mathbf{C}_{pk}$，假设其特征值$\lambda_{p1},\lambda_{p2},\lambda_{p2}$，及对应特征向量$\mathbf{u}_{p1},\mathbf{u}_{p2},\mathbf{u}_{p2}$。

$$ {}^c\mathbf{C}_{pk}^{-1} = \frac{1}{\lambda_{p1}}\mathbf{u}_{p1}\mathbf{u}_{p1}^T + \frac{1}{\lambda_{p2}}\mathbf{u}_{p2}\mathbf{u}_{p2}^T + \frac{1}{\lambda_{p3}}\mathbf{u}_{p3}\mathbf{u}_{p3}^T \quad (61) $$

对于边缘上的点，有$\lambda_{p1} \gg \lambda_{p2} \geq \lambda_{p2}$，即$\frac{1}{\lambda_{p3}} \geq \frac{1}{\lambda_{p2}} \gg \frac{1}{\lambda_{p1}}$。则

$$ J_{pk}(\xi) = \frac{1}{2}({}^c\mathbf{p}_k - T_{cr}({}^r\mathbf{p}_k,\xi))^T\,{}^c\mathbf{C}_{pk}^{-1}({}^c\mathbf{p}_k - T_{cr}({}^r\mathbf{p}_k,\xi)) $$
$$ \approx \frac{1}{2}({}^c\mathbf{p}_k - T_{cr}({}^r\mathbf{p}_k,\xi))^T\left(\frac{1}{\lambda_{p2}}\mathbf{u}_{p2}\mathbf{u}_{p2}^T + \frac{1}{\lambda_{p3}}\mathbf{u}_{p3}\mathbf{u}_{p3}^T\right)({}^c\mathbf{p}_k - T_{cr}({}^r\mathbf{p}_k,\xi)) \quad (62) $$

只有沿${}^c\mathbf{p}_k$所在边缘的垂直方向上的运动才会导致$J_{pk}(\xi)$变化。

RGB-D传感器位姿估计

基于平面和边缘点的位姿估计

参数w_{pk}的计算

先求J_{pk}在ξ处的梯度

$$\mathbf{g}_{pk} = \frac{\partial J_{pk}}{\partial \xi} = -\begin{bmatrix} \mathbf{I}_{3\times3} \\ (\mathbf{R}_{cr}{}^{r}\mathbf{p}_k)_{\times} \end{bmatrix} {}^{c}\mathbf{C}_{pk}^{-1}\left({}^{c}\mathbf{p}_k - T_{cr}\left({}^{r}\mathbf{p}_k, \xi\right)\right) \tag{63}$$

其中$(\mathbf{R}_{cr}{}^{r}\mathbf{p}_k)_{\times}$是$\mathbf{R}_{cr}{}^{r}\mathbf{p}_k$对应的反对称矩阵。令

$$\mathbf{v}_{pk} = \frac{\mathbf{g}_{pk}}{|\mathbf{g}_{pk}|} \tag{64}$$

为J_{pk}在ξ处的梯度方向。

RGB-D传感器位姿估计

基于平面和边缘点的位姿估计

参数w_{pk}的计算

$$w_{pk} = \frac{1}{6}\sum_{l=1}^{6}\frac{\left|\mathbf{v}_{pk}^T\mathbf{q}_{\pi l}\right|}{\exp\left(\alpha\sqrt{\frac{\lambda_{\pi l}}{\lambda_{\pi 1}}}\right)} \quad (65)$$

$\mathbf{v}_{pk}^T\mathbf{q}_{\pi l} \in [0,1], l = 1,\cdots,6$为$\mathbf{v}_{pk}$在$\Psi_\pi$各个主方向$\mathbf{q}_{\pi l}$上的分量，若$\mathbf{q}_{\pi l}$方向对应的$\lambda_{\pi l}$越小，其对应的分母$\exp\left(\alpha\sqrt{\frac{\lambda_{\pi l}}{\lambda_{\pi 1}}}\right) \in [1,e^\alpha]$也越小(越接近1)；若$\mathbf{q}_{\pi l}$方向对应的$\lambda_{\pi l}$越大，其对应的分母$\exp\left(\alpha\sqrt{\frac{\lambda_{\pi l}}{\lambda_{\pi 1}}}\right) \in [1,e^\alpha]$也越大(接近$e^\alpha$)。

例如：

- 若$\mathbf{v}_{pk} = \mathbf{q}_{\pi 1}$，则$w_{pk} = \frac{1}{e^\alpha}$，即在平面能提供较大约束的方向上起到抑制作用。参数α设置地越大，抑制作用越强。
- 若$\mathbf{v}_{pk} = \mathbf{q}_{\pi 6}$且$\lambda_{\pi 6} = 0$，则$w_{pk} = 1$，即在平面无法提供约束

RGB-D传感器位姿估计

基于平面和边缘点的位姿估计

参数W_p的计算

$$W_p = \frac{\beta\sqrt{\lambda_{\pi 1}}}{\left|\sum_{k=1}^{N_p} w_{pk}\mathbf{g}_{pk}\right|} \tag{66}$$

RGB-D传感器位姿估计

对应关系的建立

平面的对应关系

定义平面${}^{c}\pi_i$与${}^{r}\pi_m$之间的距离

$$D\left({}^{c}\pi_i, {}^{r}\pi_m\right) = \left({}^{c}\pi_i - {}^{r}\pi_m\right)^T {}^{c}\mathbf{C}_{\pi i}^{-1} \left({}^{c}\pi_i - {}^{r}\pi_m\right) \tag{67}$$

则对于当前帧中一个平面${}^{c}\pi_i$，其在参考帧中的对应平面${}^{r}\pi_i$为

$$ {}^{r}\pi_i = \arg\min_{{}^{r}\pi_m} D\left({}^{c}\pi_i, {}^{r}\pi_m\right) \tag{68}$$

RGB-D传感器位姿估计

对应关系的建立

平面的对应关系

剔除错配关系：

- 建立对应关系。

$$ {}^{r}\pi_i = \arg\min_{{}^{r}\pi_m} D\left({}^{c}\pi_i, {}^{r}\pi_m\right), i = 1, \cdots, {}^{c}N_\pi $$

- 计算

$$ \xi^* = \arg\min_{\xi} \sum_{i=1}^{{}^{c}N_\pi} J_{\pi i}(\xi) $$

- $\{e_i = J_{\pi i}(\xi^*), i = 1, \cdots, {}^{c}N_\pi\}$，计算其均值$\mu_e$及方差$\sigma_e^2$。
- $\forall i$，如果$e_i - \mu_e > \sigma_e$，则去除$\{{}^{c}\pi_i, {}^{r}\pi_i\}$匹配关系。

实验结果

Table: **实验一**：只用平面计算位姿，边缘点不参与计算，找出一个sequence中所有的非退化情况，计算其估计位姿结果的RPE(Relative Pose Error) RMSE，在估计平面参数的时候分别用三种方法，其他均保持一致。

	LS	LS(noise)	LS(noise&angle)
fr3/cabinet	0.0185m/1.164°	0.0131m/1.025°	**0.0124m/1.023**°
fr3/str_tex_far	0.0211m/0.873°	**0.0192m/0.782**°	0.0200m/0.803°
fr3/str_tex_near	0.0186m/1.166°	0.0148m/0.960°	**0.0117m/0.930**°
fr3/str_ntex_far	**0.0253m/1.073**°	0.0277m/1.076°	0.0280m/1.087°
fr3/str_ntex_near	0.0109m/0.769°	0.0097m/**0.768**°	**0.0095m**/0.784°

实验结果

Table: 实验二：基于平面和边缘点的位姿估计，计算其ATE(Absolute Trajectory Error) RMSE。参数设置：$\alpha = 1, \beta = 1$。

	LS	LS(noise)	LS(noise&angle)
fr1/desk	0.067m	0.044m	**0.031m**
fr1/plant	0.062m	0.047m	**0.043m**
fr2/desk	0.131m	**0.079m**	0.083m
fr3/office	0.072m	0.069m	**0.053m**
fr3/str_tex_near	0.052m	0.085m	**0.030m**
fr3/nstr_tex_near	**0.045m**	0.065m	0.058m

dataset	without plane	hard labeling	soft labeling
fr1/desk	0.034	0.080	**0.030**
fr1/plant	**0.050**	0.072	0.073
fr2/desk	0.097	0.134	**0.095**
fr3/office	0.086	0.077	**0.076**
fr3/structure_texture_near	0.049	**0.028**	0.036
fr3/nst	0.076	**0.032**	**0.032**
iclnuim/lr3	**0.002**	0.049	**0.002**
iclnuim/lr3noisy	0.028	0.024	**0.019**

Figure: Results of CPA-SLAM[3].(no final optimization.)

实验结果

Table: 实验二：基于平面和边缘点的位姿估计，计算其ATE(Absolute Trajectory Error) RMSE。参数设置：$\alpha = 1, \beta = 1$。

	0	1	2	0	1	2
fr1/desk	0.067m	0.044m	**0.031m**	0.064m	0.035m	**0.032m**
fr1/plant	0.062m	0.047m	**0.043m**	0.061m	**0.049m**	0.053m
fr2/desk	0.131m	**0.079m**	0.083m	0.106m	**0.071m**	0.087m
fr3/office	0.072m	0.069m	**0.053m**	0.069m	**0.051m**	0.059m
fr3/str_tex_near	0.052m	0.085m	**0.030m**	0.052m	0.057m	**0.051m**
fr3/nstr_tex_near	**0.045m**	0.065m	0.058m			

dataset	without plane	hard labeling	soft labeling
fr1/desk	0.034	0.080	**0.030**
fr1/plant	**0.050**	0.072	0.073
fr2/desk	0.097	0.134	**0.095**
fr3/office	0.086	0.077	**0.076**
fr3/structure_texture_near	0.049	**0.028**	0.036
fr3/nst	0.076	**0.032**	**0.032**
iclnuim/lr3	**0.002**	0.049	**0.002**
iclnuim/lr3noisy	0.028	0.024	**0.019**

Figure: Results of CPA-SLAM[4].(no final optimization.)

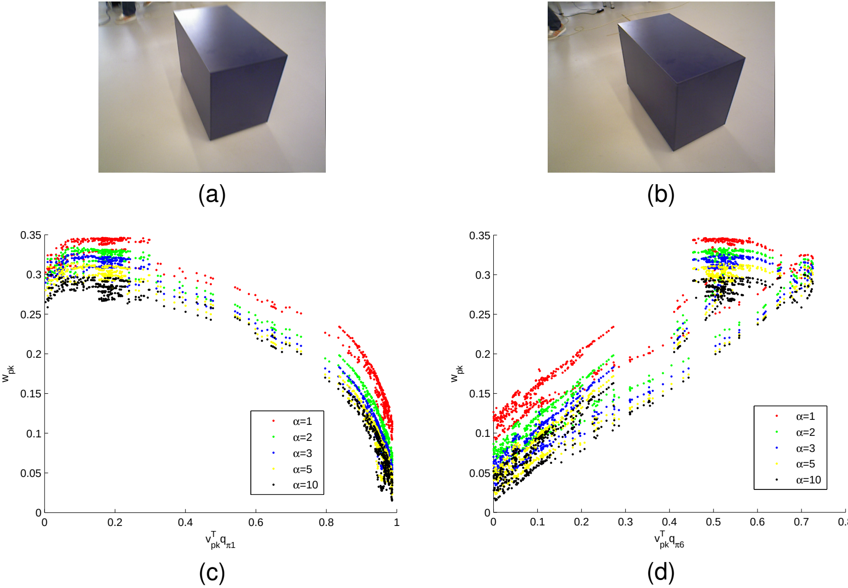

(a)
(b)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
w_{pk}
0
0.2
0.4
0.6
0.8
1
$v_{pk}^T q_{\pi 1}$
α=1
α=2
α=3
α=5
α=10
(c)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$v_{pk}^T q_{\pi 6}$
(d)

Figure:

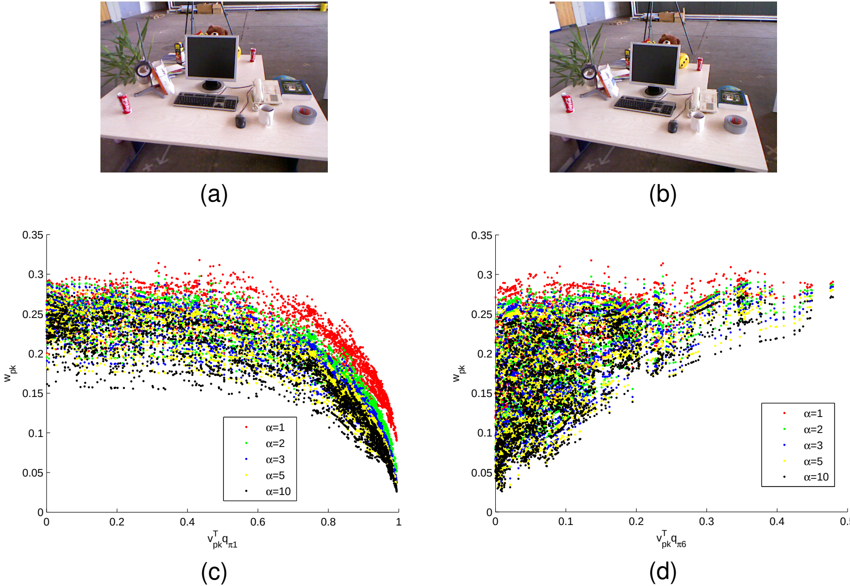
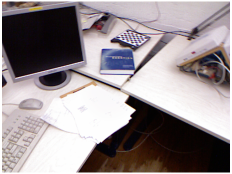
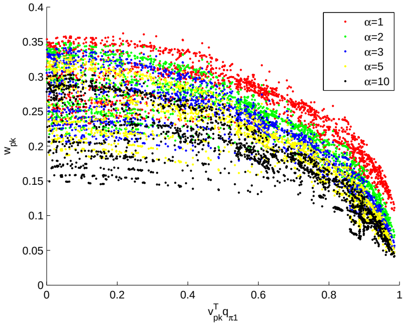

(a)
(b)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
w_{pk}
0
0.2
0.4
0.6
0.8
1
$v_{pk}^T q_{\pi 1}$
α=1
α=2
α=3
α=5
α=10
(c)
0
0.1
0.2
0.3
0.4
0.5
$v_{pk}^T q_{\pi 6}$
(d)

Figure:

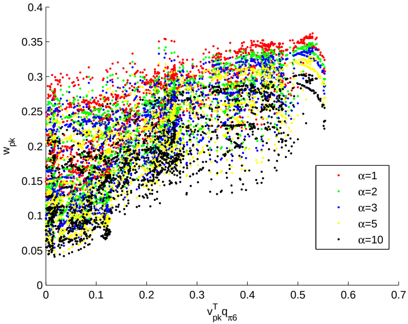


(a)

(b)


0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
w_{pk}
0
0.2
0.4
0.6
0.8
1
$v_{pk}^T q_{\pi 1}$
α=1
α=2
α=3
α=5
α=10

(c)

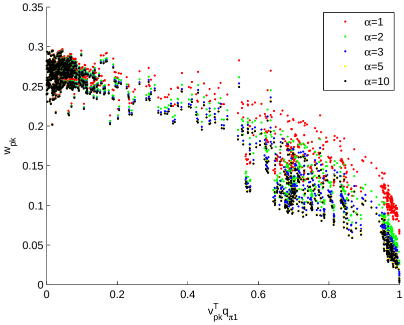

0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
w_{pk}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$v_{pk}^T q_{\pi 6}$
α=1
α=2
α=3
α=5
α=10

(d)

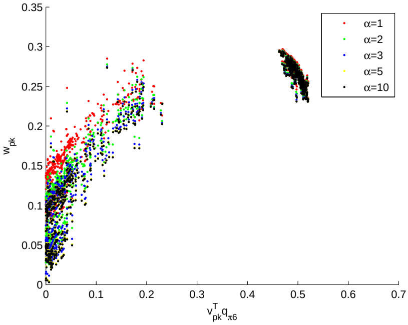
Figure:

(a)

(b)


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
w_pk
0
0.2
0.4
0.6
0.8
1
v_pk^T q_π1
α=1
α=2
α=3
α=5
α=10

(c)


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
w_pk
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
v_pk^T q_π6
α=1
α=2
α=3
α=5
α=10

(d)

Figure:

实验结果

Table: 实验三：基于平面和边缘点的位姿估计，计算其ATE(Absolute Trajectory Error) RMSE，对比加权与不加权的结果。平面参数计算方法：方法2，参数设置：$\alpha = 1, \beta = 1$。

	加权	不加权
fr1/desk	0.035m	0.062m
fr1/plant	0.049m	0.066m
fr2/desk	0.071m	0.103m
fr3/office	0.051m	0.094m
fr3/str_tex_near	0.052m	0.061m