Line-based Shadow SLAM

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OUTLINE

Plücker Coordinates of 3D Lines

- The Plücker coordinates of a 3D line are denoted by $\boldsymbol{\mathcal{L}} = [\boldsymbol{u}^T, \boldsymbol{v}^T]^T \in \mathbb{P}^5$, where the vector $\boldsymbol{u} \in \mathbb{R}^3$ is normal to the interpretation plane $\pi_{\mathcal{L}}$ containing the line $\boldsymbol{\mathcal{L}}$ and the origin, and $\boldsymbol{v} \in \mathbb{R}^3$ is the line direction. [1]
- $\boldsymbol{\mathcal{L}}$ satisfies the Plücker constraint $\boldsymbol{u}^T\boldsymbol{v} = 0$.
- The Plücker matrix is defined by

$$\boldsymbol{L} = \begin{bmatrix} [\boldsymbol{u}]_\times & \boldsymbol{v} \\ -\boldsymbol{v}^T & 0 \end{bmatrix} \tag{1}$$

[1] G. Zhang, J. Lee, J. Lim and H. Suh, Building a 3-D line-based map using stereo SLAM, IEEE Transactions on Robotics, vol. 31, no. 6, pp. 1364-1377, 2015.

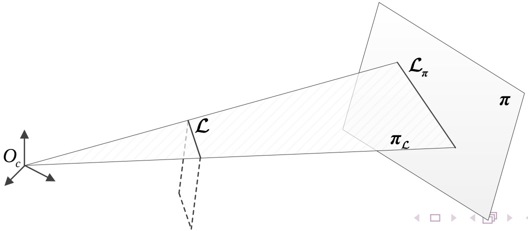
Shadow of 3D lines on Planes

- The plane $\boldsymbol{\pi}_{\boldsymbol{\mathcal{L}}} \in \mathbb{P}^3$ joining the occluding line $\boldsymbol{\mathcal{L}}$ and the origin $\boldsymbol{O}_c$ is calculated by

$$\boldsymbol{\pi}_{\boldsymbol{\mathcal{L}}} = \boldsymbol{L}^* \cdot \tilde{\boldsymbol{O}}_c = [\boldsymbol{u}^T, 0]^T \quad (2)$$

where $\tilde{\boldsymbol{O}}_c = [0,0,0,1]^T$ is the homogeneous coordinates of the origin and $\boldsymbol{L}^*$ is the dual Plücker matrix associated with $\boldsymbol{\mathcal{L}}$ which is computed by

$$\boldsymbol{L}^* = \begin{bmatrix} [\boldsymbol{v}]_\times & \boldsymbol{u} \\ -\boldsymbol{u}^T & 0 \end{bmatrix} \quad (3)$$

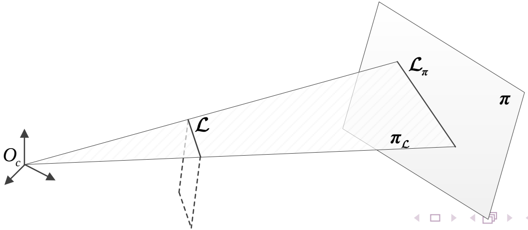


Shadow of 3D lines on Planes

- The occluded line on the plane $\boldsymbol{\pi} = [\boldsymbol{n}^T, d]^T \in \mathbb{P}^3$ corresponding to the occluding line $\mathcal{L}$ is denoted by $\mathcal{L}_\pi$. The corresponding dual Plücker matrix $\boldsymbol{L}_\pi^*$ is calculated by

$$\begin{aligned} \boldsymbol{L}_\pi^* &= \pi \pi_{\mathcal{L}}^T - \pi_{\mathcal{L}} \pi^T \\ &= \begin{bmatrix} [\boldsymbol{u} \times \boldsymbol{n}]_\times & -d\boldsymbol{u} \\ d\boldsymbol{u}^T & 0 \end{bmatrix} \end{aligned} \tag{4}$$

- The Plücker coordinates $\boldsymbol{L}_\pi^* = [-d\boldsymbol{u}^T, (\boldsymbol{u} \times \boldsymbol{n})^T]^T$.



Motion Estimation using Occluding Lines

- The left superscript c and r represent the current and reference frame, respectively.
- The rigid transformation of a plane

$$
\begin{aligned}
T_{cr}({}^{r}\pi) = \begin{bmatrix} T_{cr}({}^{r}\boldsymbol{n}) \\ T_{cr}({}^{r}\boldsymbol{n}, {}^{r}d) \end{bmatrix} &= \begin{bmatrix} \boldsymbol{R}_{cr} & \boldsymbol{0} \\ -\boldsymbol{t}_{cr}^{T}\boldsymbol{R}_{cr} & 1 \end{bmatrix} \cdot \begin{bmatrix} {}^{r}\boldsymbol{n} \\ {}^{r}\boldsymbol{d} \end{bmatrix} \\
&= \begin{bmatrix} \boldsymbol{R}_{cr}{}^{r}\boldsymbol{n} \\ -\boldsymbol{t}_{cr}^{T}\boldsymbol{R}_{cr}{}^{r}\boldsymbol{n} + {}^{r}d \end{bmatrix}
\end{aligned} \tag{5}
$$

- The rigid transformation of a 3D line

$$
\begin{aligned}
T_{cr}({}^{r}\mathcal{L}) = \begin{bmatrix} T_{cr}({}^{r}\boldsymbol{u}, {}^{r}\boldsymbol{v}) \\ T_{cr}({}^{r}\boldsymbol{v}) \end{bmatrix} &= \begin{bmatrix} \boldsymbol{R}_{cr} & [\boldsymbol{t}_{cr}]_{\times}\boldsymbol{R}_{cr} \\ \boldsymbol{0} & \boldsymbol{R}_{cr} \end{bmatrix} \cdot \begin{bmatrix} {}^{r}\boldsymbol{u} \\ {}^{r}\boldsymbol{v} \end{bmatrix} \\
&= \begin{bmatrix} \boldsymbol{R}_{cr}{}^{r}\boldsymbol{u} + [\boldsymbol{t}_{cr}]_{\times}\boldsymbol{R}_{cr}{}^{r}\boldsymbol{v} \\ \boldsymbol{R}_{cr}{}^{r}\boldsymbol{v} \end{bmatrix}
\end{aligned} \tag{6}
$$

Motion Estimation using Occluding Lines

- The objective function is defined by

$$
\begin{aligned}
E(\boldsymbol{R}_{cr},\boldsymbol{t}_{cr}) &= \alpha_\pi \sum_{j=1}^{N_\pi} \left\| {}^c\pi_j - T_{cr}({}^r\pi_j) \right\|_2^2 + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} \| {}^c\boldsymbol{\mathcal{L}}_i - T_{cr}({}^r\boldsymbol{\mathcal{L}}_i) \|_2^2 \\
&= E_1(\boldsymbol{R}_{cr}) + E_2(\boldsymbol{R}_{cr},\boldsymbol{t}_{cr})
\end{aligned} \tag{7}
$$

where

$$
E_1(\boldsymbol{R}_{cr}) = \alpha_\pi \sum_{j=1}^{N_\pi} \left\| {}^c\boldsymbol{n}_j - T_{cr}({}^c\boldsymbol{n}_j) \right\|_2^2 + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} \| {}^c\boldsymbol{v}_i - T_{cr}({}^c\boldsymbol{v}_i) \|_2^2 \tag{8}
$$

$$
E_2(\boldsymbol{R}_{cr},\boldsymbol{t}_{cr}) = \alpha_\pi \sum_{j=1}^{N_\pi} \left\| {}^c d_j - T_{cr}({}^c\boldsymbol{n}_i, {}^c d_j) \right\|^2 + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} \| {}^c\boldsymbol{u}_i - T_{cr}({}^c\boldsymbol{u}_i, {}^c\boldsymbol{v}_i) \|_2^2 \tag{9}
$$

Motion Estimation using Occluding Lines

- The rotation $\boldsymbol{R}_{cr}$ (represented by a unit quaternion $\boldsymbol{q}_{cr}$) is determined by minimizing (8).
- Rewrite (8) as

$$\begin{aligned}
E_1(\boldsymbol{R}_{cr}) &= \alpha_\pi \sum_{j=1}^{N_\pi} \left\| {}^c\boldsymbol{n}_j - T_{cr}({}^c\boldsymbol{n}_j) \right\|_2^2 + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} \| {}^c\boldsymbol{v}_i - T_{cr}({}^c\boldsymbol{v}_i) \|_2^2 \\
&= \alpha_\pi \sum_{j=1}^{N_\pi} \left\| {}^c\boldsymbol{n}_j - \boldsymbol{q}_{cr} * {}^c\boldsymbol{n}_j * \boldsymbol{q}_{cr} \right\|_2^2 + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} \| {}^c\boldsymbol{v}_i - \boldsymbol{q}_{cr} * {}^c\boldsymbol{v}_i * \boldsymbol{q}_{cr} \|_2^2 \\
&= \alpha_\pi \sum_{j=1}^{N_\pi} \left\| {}^c\boldsymbol{n}_j * \boldsymbol{q}_{cr} - \boldsymbol{q}_{cr} * {}^c\boldsymbol{n}_j \right\|_2^2 + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} \| {}^c\boldsymbol{v}_i * \boldsymbol{q}_{cr} - \boldsymbol{q}_{cr} * {}^c\boldsymbol{v}_i \|_2^2 \\
&= \alpha_\pi \sum_{j=1}^{N_\pi} \left\| \boldsymbol{A}_j \boldsymbol{q}_{cr} \right\|_2^2 + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} \| \boldsymbol{B}_i \boldsymbol{q}_{cr} \|_2^2 \\
&= \boldsymbol{q}_{cr}^T \boldsymbol{M} \boldsymbol{q}_{cr}
\end{aligned} \tag{10}$$

Motion Estimation using Occluding Lines

- with

$$\boldsymbol{M} = \alpha_\pi \sum_{j=1}^{N_\pi} \boldsymbol{A}_j^T \boldsymbol{A}_j + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} \boldsymbol{B}_i^T \boldsymbol{B}_i \in \mathbb{R}^{4\times 4}$$
$$\boldsymbol{A}_j = \begin{bmatrix} 0 & ({}^r\boldsymbol{n}_i - {}^c\boldsymbol{n}_i)^T \\ -({}^r\boldsymbol{n}_i - {}^c\boldsymbol{n}_i) & [{}^r\boldsymbol{n}_i + {}^c\boldsymbol{n}_i]_\times \end{bmatrix} \tag{11}$$
$$\boldsymbol{B}_i = \begin{bmatrix} 0 & ({}^r\boldsymbol{v}_i - {}^c\boldsymbol{v}_i)^T \\ -({}^r\boldsymbol{v}_i - {}^c\boldsymbol{v}_i) & [{}^r\boldsymbol{v}_i + {}^c\boldsymbol{v}_i]_\times \end{bmatrix}$$

- $\boldsymbol{q}_{cr}$ is the eigenvector corresponding to the smallest eigenvalue of the matrix $\boldsymbol{M}$.

Motion Estimation using Occluding Lines

- The translation $\boldsymbol{t}_{cr}$ is determined by minimizing (9).
- Letting $\frac{\partial E_2}{\partial \boldsymbol{t}_{cr}} = 0$ yields

$$\Psi \boldsymbol{t}_{cr} = \boldsymbol{b} \tag{12}$$

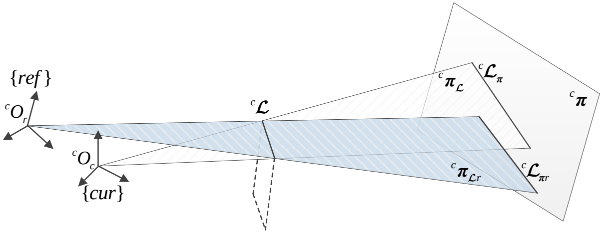
with

$$\begin{aligned} \Psi &= \alpha_\pi \sum_{j=1}^{N_\pi} (\boldsymbol{R}_{cr} {}^r\boldsymbol{n}_j)(\boldsymbol{R}_{cr} {}^r\boldsymbol{n}_j)^T + \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} [\boldsymbol{R}_{cr} {}^r\boldsymbol{u}_i]_\times^T [\boldsymbol{R}_{cr} {}^r\boldsymbol{u}_i]_\times \\ \boldsymbol{b} &= -\alpha_\pi \sum_{j=1}^{N_\pi} (\boldsymbol{R}_{cr} {}^r\boldsymbol{n}_j)({}^c d_j - {}^r d_j) - \alpha_{\mathcal{L}} \sum_{i=1}^{N_{\mathcal{L}}} [\boldsymbol{R}_{cr} {}^r\boldsymbol{u}_i]_\times^T ({}^c\boldsymbol{u}_i - \boldsymbol{R}_{cr} {}^r\boldsymbol{u}_i) \end{aligned} \tag{13}$$

Alignment of Occluded Lines

- The origin of the reference frame w.r.t. the current frame is ${}^{c}\boldsymbol{O}_r = \boldsymbol{t}_{cr}$.
- The plane ${}^{c}\pi_{\mathcal{L}r}$ joining the line ${}^{c}\boldsymbol{\mathcal{L}}$ and the origin of the reference frame ${}^{c}\boldsymbol{O}_r$ is calculated by

$$ {}^{c}\boldsymbol{\pi}_{\mathcal{L}r} = {}^{c}\boldsymbol{L}^{*} \cdot {}^{c}\tilde{\boldsymbol{O}}_r = \begin{bmatrix} [{}^{c}\boldsymbol{v}]_{\times}\boldsymbol{t}_{cr} + {}^{c}\boldsymbol{u} \\ -{}^{c}\boldsymbol{u}^{T}\boldsymbol{t}_{cr} \end{bmatrix} \quad (14) $$



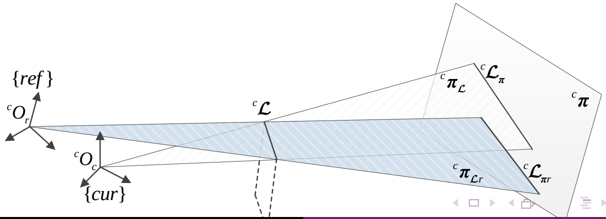
Alignment of Occluded Lines

- The occluded line ${}^{c}\mathcal{L}_{\pi r}$ intersecting ${}^{c}\pi_{\mathcal{L}r}$ and the plane ${}^{c}\pi$ is calculated by

$$\begin{aligned} {}^{c}\boldsymbol{L}^{*}_{\pi r} &= {}^{c}\pi{}^{c}\pi^{T}_{\mathcal{L}r} - {}^{c}\pi_{\mathcal{L}r}{}^{c}\pi^{T} \\ &= \begin{bmatrix} [{}^{c}\boldsymbol{v}_{\pi r}]_{\times} & {}^{c}\boldsymbol{u}_{\pi r} \\ -{}^{c}\boldsymbol{u}^{T}_{\pi r} & 0 \end{bmatrix} \end{aligned} \tag{15}$$

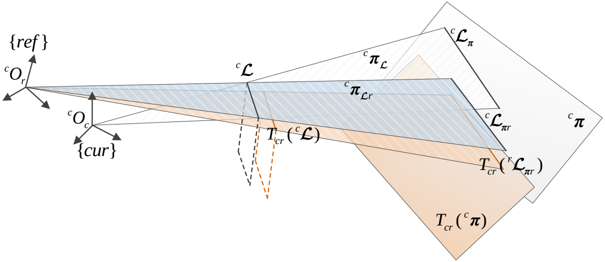
with

$$\begin{aligned} {}^{c}\boldsymbol{u}_{\pi r} &= -{}^{c}d([{}^{c}\boldsymbol{v}]_{\times}\boldsymbol{t}_{cr} + {}^{c}\boldsymbol{u}) - {}^{c}\boldsymbol{n}\boldsymbol{t}^{T}_{cr}{}^{c}\boldsymbol{u} \\ {}^{c}\boldsymbol{v}_{\pi r} &= -[{}^{c}\boldsymbol{n}]_{\times}[{}^{c}\boldsymbol{v}]_{\times}\boldsymbol{t}_{cr} + [{}^{c}\boldsymbol{u}]_{\times}{}^{c}\boldsymbol{n} \end{aligned} \tag{16}$$



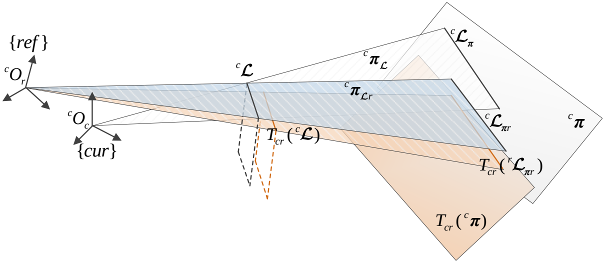
Alignment of Occluded Lines

- The measurements of the occluding line, occluded line and the plane are denoted by ${}^{r}\mathcal{L}$, ${}^{r}\mathcal{L}_{\pi r}$ and ${}^{r}\pi$, respectively (described in the reference frame).
- The transform $\boldsymbol{R}_{cr},\boldsymbol{t}_{cr}$ is refined by aligning ${}^{c}\mathcal{L}_{\pi r}$ with the measurement $T_{cr}({}^{r}\mathcal{L}_{\pi r})$.



Alignment of Occluded Lines

- ${}^{c}\boldsymbol{\mathcal{L}}_{\pi r}$ and $T_{cr}({}^{r}\boldsymbol{\mathcal{L}}_{\pi r})$ are aligned if
  - $\boldsymbol{R}_{cr}$ and $\boldsymbol{t}_{cr}$ are correctly recovered;
  - ${}^{c}\pi = T_{cr}({}^{r}\pi)$;
  - ${}^{c}\boldsymbol{\mathcal{L}} = T_{cr}({}^{r}\boldsymbol{\mathcal{L}})$.



Alignment of Occluded Lines

- The objective function is defined by

$$
\begin{aligned}
F(\boldsymbol{R}_{cr}, \boldsymbol{t}_{cr}) = \sum_{i=1}^{N_{\mathcal{L}}} \left\{ \left\| {}^{c}\boldsymbol{\mathcal{L}}_{\pi ri} - T_{cr}({}^{r}\boldsymbol{\mathcal{L}}_{\pi ri}) \right\|_2^2 + \left\| {}^{c}\boldsymbol{\mathcal{L}}_{i} - T_{cr}({}^{r}\boldsymbol{\mathcal{L}}_{i}) \right\|_2^2 \right\} \\
+ \sum_{j=1}^{N_{\pi}} \left\| {}^{c}\pi_j - T_{cr}({}^{r}\pi_j) \right\|_2^2
\end{aligned}
\tag{17}
$$