Plane Feature and Projective Shadow based SLAM

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July 10, 2017

1 Camera Tracking

There are N_π pairs of matched planes $\{{}^c\pi_i, {}^r\pi_i\}_{i=1,\dots,N}$ between two frames (or between the map and one frame). $\left\{{}^c\mathbf{p}^i_{s,k}, {}^c\mathbf{p}^i_{o,k}\right\}_{k=1,\dots,N^i_p}$ denote shadow points ${}^c\mathbf{p}^i_{s,k}$ on plane ${}^c\pi_i$ with corresponding occluding points ${}^c\mathbf{p}^i_{o,k}$ causing the shadow.

$\mathbf{w} = [\mathbf{t}^T, \omega^T]^T$ denotes the 6-DoF camera pose.

i	plane index;
k	point index;
c presuperscript	current frame;
r presuperscript	reference frame;
s subscript	shadow point;
o subscript	occluding point;
$\pi_i = [\mathbf{n}_i^T, d_i]^T$	parameters of the i-th plane;
$\mathbf{p}^i_{s,k}$	k-th shadow points on the i-th plane;
$\mathbf{p}^i_{o,k}$	the occluding point corresponding to ${}^c p^i_{s,k}$;
$\{{}^c\pi_i, {}^r\pi_i\}_{i=1,\dots,N_\pi}$	matched planes;
$\left\{{}^c\mathbf{p}^i_{s,k}, {}^c\mathbf{p}^i_{o,k}\right\}_{k=1,\dots,N^i_p}$	shadow points and corresponding occluding points;
$\mathbf{w} = [\mathbf{t}^T, \omega^T]^T$	6-DoF camera pose;
$T_{cr} \in \mathbb{SE}(3)$	transformation from reference to current frame;
$\mathbf{R}_{cr} \in \mathbb{SO}(3)$	rotation matrix;
$\mathbf{t}_{cr} \in \mathbb{R}^3$	translation vector;

The overall objective function is

$$J(\mathbf{w}) = J_\pi(\mathbf{w}) + J_p(\mathbf{w}) \tag{1}$$

$$J_\pi(\mathbf{w}) = \sum_{i=1}^{N} J_{\pi,i} = \sum_{i=1}^{N} \left(\|{}^c\mathbf{n}_i - \mathbf{R}_{cr}{}^r\mathbf{n}_i\|^2 + \left[{}^c d_i - \left({}^r d_i + {}^c\mathbf{n}_i^T \mathbf{t}_{cr}\right)\right]^2 \right) \tag{2}$$

$$J_p(\mathbf{w}) = \sum_{i=1}^{N}\sum_{k=1}^{N^i_p} J^i_{p,k} = \sum_{i=1}^{N}\sum_{k=1}^{N^i_p} {\varepsilon^i_k}^T \varepsilon^i_k = \sum_{i=1}^{N}\sum_{k=1}^{N^i_p} \left({}^c\mathbf{p}^i_{s,k} - {}^c\mathbf{p}^i_{proj,k}\right)^T \left({}^c\mathbf{p}^i_{s,k} - {}^c\mathbf{p}^i_{proj,k}\right) \tag{3}$$

$${}^c\mathbf{p}^i_{proj,k} = -\frac{{}^c d_i}{{}^c\mathbf{n}_i^T T_{cr}({}^r\mathbf{p}^i_{o,k})} T_{cr}({}^r\mathbf{p}^i_{o,k}) \tag{4}$$

1.1 Plane Parameters' Constraints on Camera Motion

$$\begin{aligned}\boldsymbol{\Psi}_{\pi} &= \sum_{i=1}^{N} \left(\frac{\partial J_{\pi,i}}{\partial \mathbf{w}}\right)\left(\frac{\partial J_{\pi,i}}{\partial \mathbf{w}}\right)^{T} \\ &= \sum_{i=1}^{N} 4 \begin{bmatrix} \left({}^{r}d_i - {}^{c}d_i\right)^2 {}^{c}\mathbf{n}_i {}^{c}\mathbf{n}_i^T & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \left({}^{c}\mathbf{n}_i \times {}^{r}\mathbf{n}_i\right)\left({}^{c}\mathbf{n}_i \times {}^{r}\mathbf{n}_i\right)^T \end{bmatrix}\end{aligned} \tag{5}$$

The matrix Ψ_π is actually a scatter matrix which contains information about the distribution of the gradient of $J_{\pi,i}$ w.r.t. $\mathbf{w}$ over all planes in the matched plane set. Performing principal component analysis upon Ψ_π results in

$$\Psi_\pi = Q_\pi \Lambda_\pi Q_\pi^T = \begin{bmatrix}\mathbf{q}_{\pi 1} & \cdots & \mathbf{q}_{\pi 6}\end{bmatrix} \begin{bmatrix} \lambda_{\pi 1} & & \\ & \ddots & \\ & & \lambda_{\pi 6}\end{bmatrix} \begin{bmatrix}\mathbf{q}_{\pi 1}^T \\ \vdots \\ \mathbf{q}_{\pi 6}^T\end{bmatrix} \tag{6}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_6$ are the eigenvalues of Ψ_π, and $\mathbf{q}_i$ are the corresponding eigenvectors, of which the first three elements are the translation components, and the last three elements are the rotation components. The eigenvector $\mathbf{q}_1$ corresponding to the largest eigenvalue represents the transformation of maximum constraint. Perturbing the plane parameters by the transformation of the direction $\mathbf{q}_1$ will result in the largest possible change in from among all possible transformation perturbations.

1.2 Shadow Points' Constraints on Camera Motion

$$\boldsymbol{\Psi}_p = \sum_{i=1}^{N} \sum_{k=1}^{N_p^i} \left(\frac{\partial J_{p,k}^i}{\partial \mathbf{w}}\right)\left(\frac{\partial J_{p,k}^i}{\partial \mathbf{w}}\right)^T \tag{7}$$

$$\frac{\partial J_{p,k}^i}{\partial \mathbf{w}} = -\frac{{}^{c}d_i}{{}^{c}\mathbf{n}_i^T {}^{r}\mathbf{p}_{o,k}^i} \begin{bmatrix}\mathbf{I}_{3\times 3} \\ {}^{r}\hat{\mathbf{p}}_{o,k}^i\end{bmatrix} \begin{bmatrix}\mathbf{I}_{3\times 3} - \frac{{}^{c}\mathbf{n}_i {}^{r}\mathbf{p}_{o,k}^i}{{}^{c}\mathbf{n}_i^T {}^{r}\mathbf{p}_{o,k}^i}\end{bmatrix}^T \left({}^{c}\mathbf{p}_{s,k}^i - {}^{c}\mathbf{p}_{proj,k}^i\right) \tag{8}$$

1.3 Shadow Point Weighting

For each shadow point ${}^{c}\mathbf{p}_{s,k}^i$, a weight $\alpha_{p,k}^i$ is assigned to the corresponding component in the cost function

$$J_p = \sum_{i=1}^{N} \sum_{k=1}^{N_p^i} \alpha_{p,k}^i J_{p,k}^i \tag{9}$$

$$\alpha_{p,k}^i = \frac{\left(\frac{\partial J_{p,k}^i}{\partial \mathbf{w}}\right)^T \mathbf{q}_{\pi,j}}{\left|\frac{\partial J_{p,k}^i}{\partial \mathbf{w}}\right| \sqrt{\frac{\lambda_{\pi,j}}{\bar{\lambda}_\pi}}} \tag{10}$$

$$j = \arg\max_j \frac{\left(\frac{\partial J_{p,k}^i}{\partial \mathbf{w}}\right)^T \mathbf{q}_{\pi,j}}{\left|\frac{\partial J_{p,k}^i}{\partial \mathbf{w}}\right|} \tag{11}$$

$$\bar{\lambda}_\pi = \frac{1}{6} \sum_{t=1}^{6} \lambda_{\pi,t} \tag{12}$$

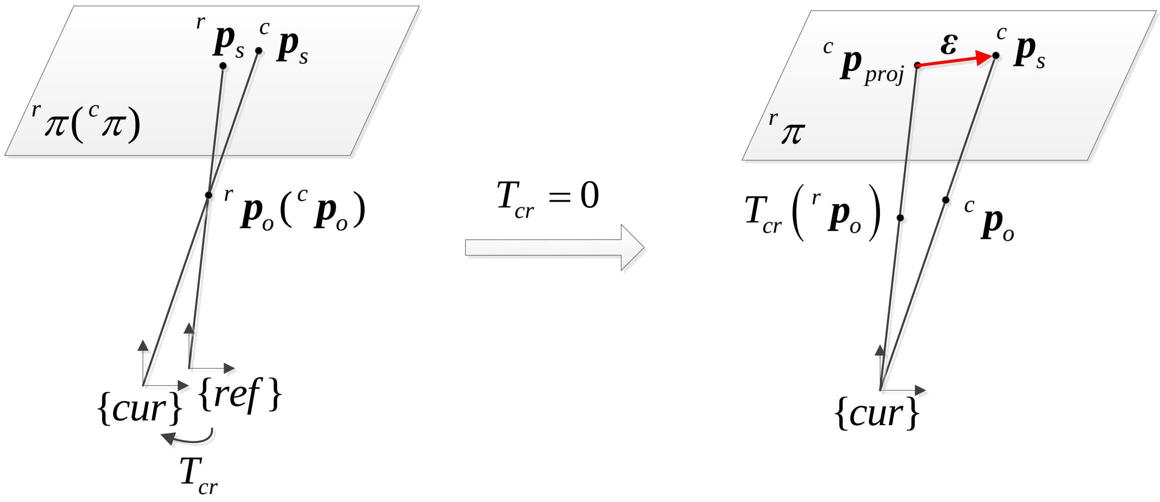

Figure 1:

2 Plane Fusion

A plane $\mathcal{P}_i$

- plane parameters $\pi_i = [\mathbf{n}_i^T, d_i]^T$
- number of points $N_{\pi,i}$
- centroid $\mathbf{p}_{\pi,i}$
- covariance $\mathbf{C}_{\pi,i}$
- curvature $\rho_{\pi,i}$ [1]
- shadow points $\left\{\mathbf{p}_{s,k}^i, k = 1, \cdots, N_p^i\right\}$

2.1 Plane Fusion

Assuming that $\mathcal{P}_i$ and $\mathcal{P}_j$ need to be merged, the number of points is the sum of the point numbers of the two components, i.e., $N = N_{\pi,i} + N_{\pi,j}$. The centroid and covariance of the merged plane $\mathcal{P}$ are computed by (13) and (14).

$$\mathbf{p}_\pi = \frac{N_{\pi,i}\mathbf{p}_{\pi,i} + N_{\pi,j}\mathbf{p}_{\pi,j}}{N_{\pi,i} + N_{\pi,j}} \tag{13}$$

$$\mathbf{C}_\pi = \frac{N_{\pi,i}(\mathbf{C}_{\pi,i} + \mathbf{p}_{\pi,i}\mathbf{p}_{\pi,i}^T) + N_{\pi,j}(\mathbf{C}_{\pi,j} + \mathbf{p}_{\pi,j}\mathbf{p}_{\pi,j}^T)}{N_{\pi,i} + N_{\pi,j}} - \mathbf{p}_\pi\mathbf{p}_\pi^T \tag{14}$$

The plane parameters $\pi = [\mathbf{n}^T, d]^T$ and the curvature ρ_π can be computed using the fused centroid and covariance matrix. Let λ_{min} and $\mathbf{q}_{min}$ be the smallest eigenvalue of

[1]Note that the curvature here is just an indication that tells how a surface deviates from being a flat plane, rather than the strictly defined curvature.

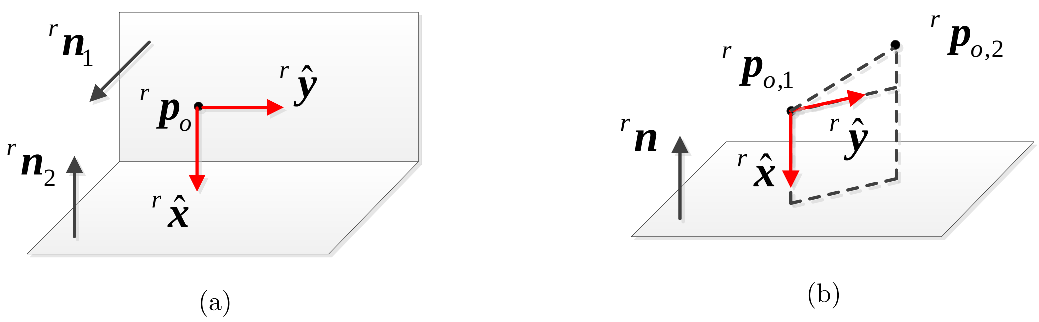

Figure 2:

$\mathbf{C}_\pi$ and the corresponding eigenvector.

$$\begin{aligned} \mathbf{n} &= \mathbf{q}_{min} \\ d &= -\mathbf{p}_\pi^T \mathbf{n} \\ \rho_\pi &= \frac{\lambda_{min}}{\text{trace}(\mathbf{C}_\pi)} \end{aligned} \tag{15}$$

3 Least Primitives for Pose Estimation

3.1 Three Planes

Three corresponding non-parallel planes $\{{}^r\pi_i, {}^c\pi_i,\}_{i=1,2,3}$. Let

$$\begin{aligned} {}^r\mathbf{M}_1 &= \begin{bmatrix} {}^r\mathbf{n}_1 & {}^r\mathbf{n}_2 & {}^r\mathbf{n}_3 \end{bmatrix} \\ {}^c\mathbf{M}_1 &= \begin{bmatrix} {}^c\mathbf{n}_1 & {}^c\mathbf{n}_2 & {}^c\mathbf{n}_3 \end{bmatrix} \\ \mathbf{d} &= \begin{bmatrix} {}^cd_1 - {}^rd_1 \\ {}^cd_2 - {}^rd_2 \\ {}^cd_3 - {}^rd_3 \end{bmatrix} \end{aligned} \tag{16}$$

The rotation $\mathbf{R}_{cr}$ and translation $\mathbf{t}_{cr}$ can be computed as

$$\begin{aligned} \mathbf{R}_{cr} &= {}^c\mathbf{M}_1 {}^r\mathbf{M}_1^{-1} \\ \mathbf{t}_{cr} &= {}^c\mathbf{M}_1^{-T} \mathbf{d} \end{aligned} \tag{17}$$

3.2 Two Planes and One Point

Two corresponding non-parallel planes $\{{}^r\pi_i, {}^c\pi_i,\}_{i=1,2}$ and one corresponding point $\{{}^r\mathbf{p}_o, {}^c\mathbf{p}_o\}$. Construct three axes ${}^r\hat{\mathbf{x}}, {}^r\hat{\mathbf{y}}, {}^r\hat{\mathbf{z}}$ in reference coordinate system and locate the origin at ${}^r\mathbf{p}_o$.

$$\begin{aligned} {}^r\hat{\mathbf{x}} &= -{}^r\mathbf{n}_2 \\ {}^r\hat{\mathbf{y}} &= {}^r\mathbf{n}_1 \times {}^r\mathbf{n}_2 \\ {}^r\hat{\mathbf{z}} &= {}^r\hat{\mathbf{x}} \times {}^r\hat{\mathbf{y}} \end{aligned} \tag{18}$$

The axes and the origin in current frame is constructed likewise.

$$\begin{aligned} {}^c\hat{\mathbf{x}} &= -{}^c\mathbf{n}_2 \\ {}^c\hat{\mathbf{y}} &= {}^c\mathbf{n}_1 \times {}^c\mathbf{n}_2 \\ {}^c\hat{\mathbf{z}} &= {}^c\hat{\mathbf{x}} \times {}^c\hat{\mathbf{y}} \end{aligned} \tag{19}$$

Let

$$
\begin{aligned}
{}^{r}\mathbf{M}_2 &= \begin{bmatrix} {}^{r}\hat{\mathbf{x}} & {}^{r}\hat{\mathbf{y}} & {}^{r}\hat{\mathbf{z}} \end{bmatrix} \\
{}^{c}\mathbf{M}_2 &= \begin{bmatrix} {}^{c}\hat{\mathbf{x}} & {}^{c}\hat{\mathbf{y}} & {}^{c}\hat{\mathbf{z}} \end{bmatrix}
\end{aligned} \tag{20}
$$

Then the rotation $\mathbf{R}_{cr}$ and translation $\mathbf{t}_{cr}$ can be computed as

$$
\begin{aligned}
\mathbf{R}_{cr} &= {}^{c}\mathbf{M}_2 {}^{r}\mathbf{M}_2^T \\
\mathbf{t}_{cr} &= {}^{c}\mathbf{p}_o - \mathbf{R}_{cr} {}^{r}\mathbf{p}_o
\end{aligned} \tag{21}
$$

3.3 One Plane and Two Points

One corresponding plane $\{{}^{r}\pi, {}^{c}\pi, \}$ and two different corresponding points $\{{}^{r}\mathbf{p}_{o,j}, {}^{c}\mathbf{p}_{o,j}\}_{j=1,2}$ satisfying $({}^{r}\mathbf{p}_{o,1} - {}^{r}\mathbf{p}_{o,2}) \times {}^{r}\mathbf{n} \neq 0$ and $({}^{c}\mathbf{p}_{o,1} - {}^{c}\mathbf{p}_{o,2}) \times {}^{c}\mathbf{n} \neq 0$. Construct three axes ${}^{r}\hat{\mathbf{x}}, {}^{r}\hat{\mathbf{y}}, {}^{r}\hat{\mathbf{z}}$ in reference coordinate system and locate the origin at ${}^{r}\mathbf{p}_{o,1}$.

$$
\begin{aligned}
{}^{r}\hat{\mathbf{x}} &= -{}^{r}\mathbf{n} \\
{}^{r}\mathbf{y} &= ({}^{r}\mathbf{p}_{o,2} - {}^{r}\mathbf{p}_{o,1}) - \left(({}^{r}\mathbf{p}_{o,2} - {}^{r}\mathbf{p}_{o,1})^T \, {}^{r}\mathbf{n} \right) {}^{r}\mathbf{n} \\
{}^{r}\hat{\mathbf{y}} &= \frac{{}^{r}\mathbf{y}}{\|{}^{r}\mathbf{y}\|} \\
{}^{r}\hat{\mathbf{z}} &= {}^{r}\hat{\mathbf{x}} \times {}^{r}\hat{\mathbf{y}}
\end{aligned} \tag{22}
$$

The axes and the origin in current frame is constructed likewise.

$$
\begin{aligned}
{}^{c}\hat{\mathbf{x}} &= -{}^{c}\mathbf{n} \\
{}^{c}\mathbf{y} &= ({}^{c}\mathbf{p}_{o,2} - {}^{c}\mathbf{p}_{o,1}) - \left(({}^{c}\mathbf{p}_{o,2} - {}^{c}\mathbf{p}_{o,1})^T \, {}^{c}\mathbf{n} \right) {}^{c}\mathbf{n} \\
{}^{c}\hat{\mathbf{y}} &= \frac{{}^{c}\mathbf{y}}{\|{}^{c}\mathbf{y}\|} \\
{}^{c}\hat{\mathbf{z}} &= {}^{c}\hat{\mathbf{x}} \times {}^{c}\hat{\mathbf{y}}
\end{aligned} \tag{23}
$$

Let

$$
\begin{aligned}
{}^{r}\mathbf{M}_3 &= \begin{bmatrix} {}^{r}\hat{\mathbf{x}} & {}^{r}\hat{\mathbf{y}} & {}^{r}\hat{\mathbf{z}} \end{bmatrix} \\
{}^{c}\mathbf{M}_3 &= \begin{bmatrix} {}^{c}\hat{\mathbf{x}} & {}^{c}\hat{\mathbf{y}} & {}^{c}\hat{\mathbf{z}} \end{bmatrix}
\end{aligned} \tag{24}
$$

Then the rotation $\mathbf{R}_{cr}$ and translation $\mathbf{t}_{cr}$ can be computed as

$$
\begin{aligned}
\mathbf{R}_{cr} &= {}^{c}\mathbf{M}_3 {}^{r}\mathbf{M}_3^T \\
\mathbf{t}_{cr} &= {}^{c}\mathbf{p}_{o,1} - \mathbf{R}_{cr} {}^{r}\mathbf{p}_{o,1}
\end{aligned} \tag{25}
$$