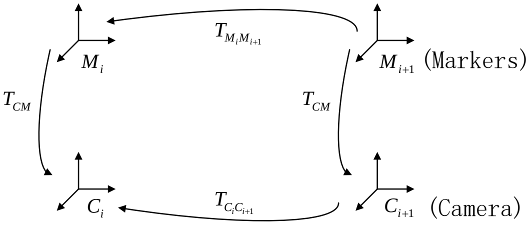
OUTLINE

Extrinsic Calibration of Qualysis and Kinect 2.0



$$T_{CM}T_{M_iM_{i+1}} = T_{C_iC_{i+1}}T_{CM} \tag{1}$$

Rewritten Eq.(1) as

$$\boldsymbol{R}_{CM}\boldsymbol{R}_{M_iM_{i+1}} = \boldsymbol{R}_{C_iC_{i+1}}\boldsymbol{R}_{CM} \tag{2}$$

$$\boldsymbol{R}_{CM}\boldsymbol{t}_{M_iM_{i+1}} + \boldsymbol{t}_{CM} = \boldsymbol{R}_{C_iC_{i+1}}\boldsymbol{t}_{CM} + \boldsymbol{t}_{C_iC_{i+1}} \tag{3}$$

Extrinsic Calibration of Qualysis and Kinect 2.0

- Solve for rotation $\boldsymbol{R}_{CM}$

$$\boldsymbol{R}_{CM}\boldsymbol{R}_{M_iM_{i+1}} = \boldsymbol{R}_{C_iC_{i+1}}\boldsymbol{R}_{CM} \tag{4}$$

- Represent the rotation by unit quaternions

$$\boldsymbol{q}_{CM}\boldsymbol{q}_{M_iM_{i+1}} = \boldsymbol{q}_{C_iC_{i+1}}\boldsymbol{q}_{CM} \tag{5}$$

$$\mathring{r}\mathring{q} = \begin{bmatrix} r_0 & -r_x & -r_y & -r_z \\ r_x & r_0 & -r_z & r_y \\ r_y & r_z & r_0 & -r_x \\ r_z & -r_y & r_x & r_0 \end{bmatrix} \mathring{q} = \mathbb{R}\,\mathring{q}$$

$$\mathring{q}\mathring{r} = \begin{bmatrix} r_0 & -r_x & -r_y & -r_z \\ r_x & r_0 & r_z & -r_y \\ r_y & -r_z & r_0 & r_x \\ r_z & r_y & -r_x & r_0 \end{bmatrix} \mathring{q} = \bar{\mathbb{R}}\,\mathring{q}.$$

Extrinsic Calibration of Qualysis and Kinect 2.0

- Solve for rotation $\boldsymbol{R}_{CM}$

$$\boldsymbol{R}_{CM}\boldsymbol{R}_{M_iM_{i+1}} = \boldsymbol{R}_{C_iC_{i+1}}\boldsymbol{R}_{CM} \tag{6}$$

- Represent the rotation by unit quaternions

$$\boldsymbol{q}_{CM}\boldsymbol{q}_{M_iM_{i+1}} = \boldsymbol{q}_{C_iC_{i+1}}\boldsymbol{q}_{CM} \tag{7}$$

$$\bar{\boldsymbol{Q}}_{M_iM_{i+1}}\boldsymbol{q}_{CM} = \boldsymbol{Q}_{C_iC_{i+1}}\boldsymbol{q}_{CM} \tag{8}$$

$$\left(\bar{\boldsymbol{Q}}_{M_iM_{i+1}} - \boldsymbol{Q}_{C_iC_{i+1}}\right)\boldsymbol{q}_{CM} = 0 \tag{9}$$

- Letting $\boldsymbol{Q}_i = \bar{\boldsymbol{Q}}_{M_iM_{i+1}} - \boldsymbol{Q}_{C_iC_{i+1}}$, and given $N+1$ observed poses,

$$\begin{bmatrix} \boldsymbol{Q}_1 \\ \boldsymbol{Q}_2 \\ \vdots \\ \boldsymbol{Q}_N \end{bmatrix} \boldsymbol{q}_{CM} = \boldsymbol{Q}\boldsymbol{q}_{CM} = 0 \tag{10}$$

- $\text{rank}(\boldsymbol{Q}) = 3$ and $\boldsymbol{q}_{CM} = \text{null}(\boldsymbol{Q})$.

Extrinsic Calibration of Qualysis and Kinect 2.0

- Solve for translation $\boldsymbol{t}_{CM}$

$$\boldsymbol{R}_{CM}\boldsymbol{t}_{M_iM_{i+1}} + \boldsymbol{t}_{CM} = \boldsymbol{R}_{C_iC_{i+1}}\boldsymbol{t}_{CM} + \boldsymbol{t}_{C_iC_{i+1}} \tag{11}$$

$$(\boldsymbol{R}_{C_iC_{i+1}} - \boldsymbol{I})\boldsymbol{t}_{CM} = \boldsymbol{R}_{CM}\boldsymbol{t}_{M_iM_{i+1}} - \boldsymbol{t}_{C_iC_{i+1}} \tag{12}$$

Continuous Homography Constraint

- $\lambda \boldsymbol{x} = \boldsymbol{X}$
- $\dot{\boldsymbol{X}} = \dot{\lambda}\boldsymbol{x} + \lambda\dot{\boldsymbol{x}} = \dot{\lambda}\boldsymbol{x} + \lambda\boldsymbol{u}$

$$\boldsymbol{u} = \boldsymbol{H}\boldsymbol{x} - \frac{\dot{\lambda}}{\lambda}\boldsymbol{x} \tag{13}$$

- Continuous homography constraint

$$\hat{\boldsymbol{x}}\boldsymbol{H}\boldsymbol{x} = \hat{\boldsymbol{x}}\boldsymbol{u} \tag{14}$$

Extended 4-Point Algorithm

- Continuous homography constraint

$$\hat{\boldsymbol{x}}\boldsymbol{H}\boldsymbol{x} = \hat{\boldsymbol{x}}\boldsymbol{u} \tag{15}$$

- Stack the elements of $\boldsymbol{H}$ into the vector $\boldsymbol{H}^S = [H_{11}, H_{21}, \cdots, H_{33}] \in \mathbb{R}^9$ and rewrite (15) as

$$\boldsymbol{a}^T \boldsymbol{H}^S = \hat{\boldsymbol{x}}\boldsymbol{u} \tag{16}$$

- Stack all the $\boldsymbol{a}_i$ from n tracked features into $\boldsymbol{A} = [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_n]^T \in \mathbb{R}^{3n \times 9}$ and stack all $\hat{\boldsymbol{x}}_i \boldsymbol{u}_i$ into $\boldsymbol{B} = [\hat{\boldsymbol{x}}_1 \boldsymbol{u}_1, \cdots, \hat{\boldsymbol{x}}_n \boldsymbol{u}_n]^T \in \mathbb{R}^{3n}$.

$$\boldsymbol{A}\boldsymbol{H}^S = \boldsymbol{B} \tag{17}$$

Extended 4-Point Algorithm

- The angular velocity is obtained directly from the IMU $\boldsymbol{\omega} = \boldsymbol{\omega}_{IMU}$.
- Subtract the rotational components from the perceived flow using the interaction matrix which relates $\boldsymbol{u}$ to $(\boldsymbol{v}, \boldsymbol{\omega})$.

$$\begin{bmatrix} u'_x \\ u'_y \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} - \begin{bmatrix} -x_x x_y & 1 + x_x^2 & -x_y \\ -(1+x_y)^2 & x_x x_y & x_x \end{bmatrix} \boldsymbol{\omega} \tag{18}$$

- Thus $\boldsymbol{H}$ reduces to

$$\boldsymbol{H} = \frac{1}{d} \boldsymbol{v} \boldsymbol{N}^T \tag{19}$$

- $\boldsymbol{N}$ spans $\boldsymbol{H}^T$ and $\|\boldsymbol{N}\| = 1$ and so $\boldsymbol{N}$ can be obtained by an SVD of $\boldsymbol{H}$.
- $\boldsymbol{v}/d = \boldsymbol{H}\boldsymbol{N}$.

Planarity Measures

- To test whether a certain group of observed features belongs to a common plane.
- Consider two different quantitative measures.
- First one,
  - $\boldsymbol{A}\boldsymbol{H}^S = \boldsymbol{B}$
  - $\text{rank}(\boldsymbol{A}) = \text{rank}([\boldsymbol{A}\ \boldsymbol{B}]) = 8$
  - $\sigma_i, i = 1, \cdots, 10$ are the singular values of $[\boldsymbol{A}\ \boldsymbol{B}] \in \mathbb{R}^{3n \times 10}$ in decreasing order.
  - The value σ_9 can be exploited as a measure of how well a certain set of features/optical flow meets the planarity constraint.
- Second one,
  - $\boldsymbol{H}^S = \boldsymbol{A}^\dagger \boldsymbol{B}$ is the LS solution.
  - Consider the "reprojection" vector

$$\boldsymbol{R} = \boldsymbol{B} - \boldsymbol{A}\boldsymbol{H}^S = \boldsymbol{B} - \boldsymbol{A}\boldsymbol{A}^\dagger \boldsymbol{B} = (\boldsymbol{I} - \boldsymbol{A}\boldsymbol{A}^\dagger)\boldsymbol{B} \tag{20}$$

  - another measure $r = \|\boldsymbol{R}\|/n$